Digital Communication Systems ECS 452

Asst. Prof. Dr. Prapun Suksompong

prapun@siit.tu.ac.th

Fading Channels

Office Hours:

Rangsit Library:

Tuesday 16:20-17:20

BKD3601-7:

Thursday 16:00-17:00

Problems of Wireless Comm.

- **Impairment: Multipath**-induced **fading**
  - **Fading** = random fluctuation in signal level   to fade = to fluctuate randomly.
  - The arrival of the transmitted signal at an intended receiver through differing angles and/or differing time delays and/or differing frequency (i.e., Doppler) shifts due to the scattering of electromagnetic waves in the environment.
    - Transmitted signals are received through **multiple paths** which usually add **destructively**
  - Consequently, the received signal power fluctuates in space (due to angle spread) and/or frequency (due to delay spread) and/or time (due to Doppler spread) through the **random superposition** of the impinging multi-path components.
- Recource constraints/scarcity:
  - Limited **power**
    - Highly constrained transmit powers
  - Scarce frequency **bandwidth** (radio spectrum)
- Unlike wireline communications, in which capacity can be increased by *adding infrastructure* such as new optical fiber,
  - wireless capacity increases have traditionally required increases in either the radio bandwidth or power, both of which are severely limited in most wireless systems.
- **Interference**: Information is transmitted not by a single source but by several (uncoordinated, bursty, and geographically separated) sources/users/applications.

~~Bad~~ Naive solution to improve BW efficiency

- How to transmit more using the same amount of BW?
- **Simple/naive approach** that naturally comes to mind: use **higher order modulation schemes**.
  - Drawback: **poor reliability**
    - For the same level of transmit power, higher order modulation schemes yield performance that is inferior to that of the lower order modulation schemes.
    - In fact, even for small signal constellations, i.e., low-order modulation schemes (e.g. binary), the reliability of uncoded communications over wireless links is very poor in general.
- Multiantenna systems offer such a possibility.

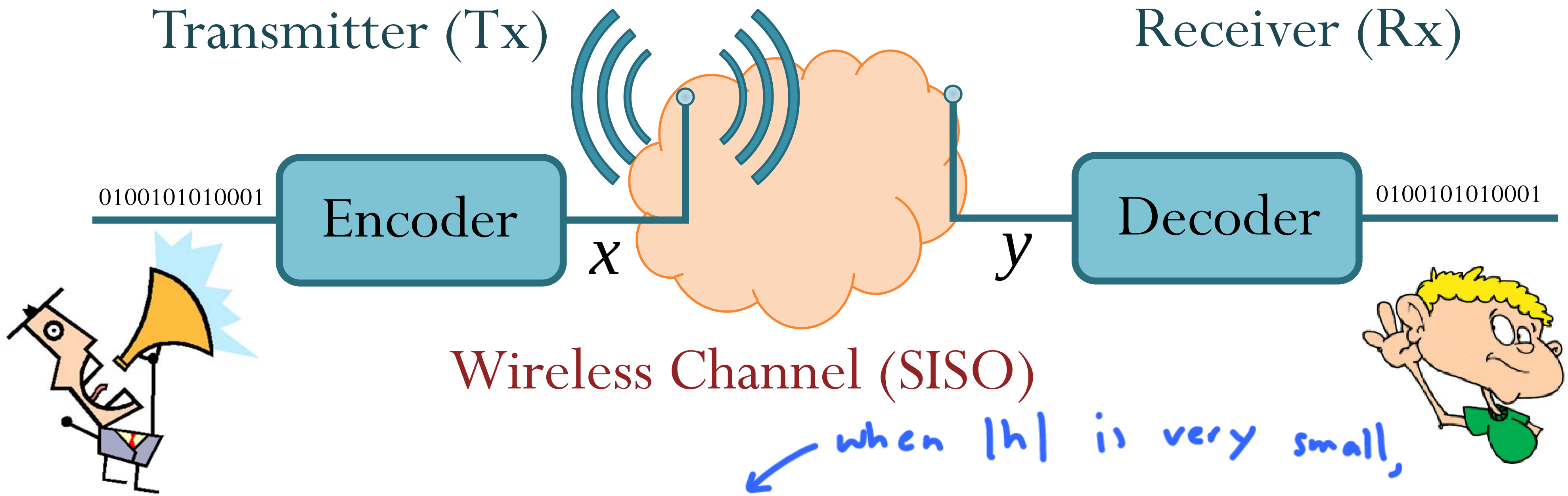
Better Solutions

- The single most effective technique to accomplish reliable communication over a wireless channel is **diversity** which
  - attempts to provide the receiver with independently faded copies of the transmitted signal
  - with the hope that at least one of these replicas will be received correctly.
- Diversity may be realized in different ways, including
  - **frequency** diversity,
  - **time** (temporal) diversity,
  - (transmit and/or receive) antenna diversity (**spatial** diversity),
  - modulation diversity, etc.
- **Channel coding** may also be used to provide (a form of time) diversity for immunization against the impairments of the wireless channel.
  - In the context of wireless communications, channel coding schemes are usually combined with **interleaving** to achieve time diversity in an efficient manner.

New View

- While channel fading has **traditionally** been regarded as a source of **unreliability** that has to be mitigated, information theory and channel capacity analysis have suggested an opposite view:
- Channel fading can instead be **exploited**.

Wireless Digital Comm. System



$$y = hx + n$$

Channel gain

(Channel (fading) coefficient)

Noise

when $|h|$ is very small, we say the system is in "deep fade".

Probability Facts

- Consider a complex-valued RV

$$Z = X + jY \quad \text{where} \quad X, Y \overset{\text{i.i.d.}}{\sim} \mathcal{N}\left(0, \sigma^2\right)$$

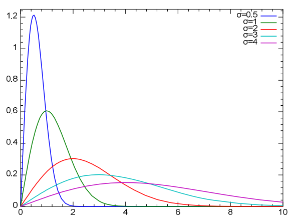
- Let R and Θ be the magnitude and phase of the RV above.
- Then
  1. R and Θ are independent.
  2. Θ is uniformly distributed on $[0, 2\pi]$
  3. R has a **Rayleigh** pdf:

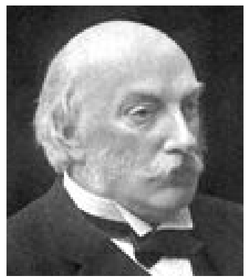
(Read: ray'-lee)

$$F_R(r) = \begin{cases} 1 - e^{-\frac{1}{2}\left(\frac{r}{\sigma}\right)^2}, & r > 0, \\ 0, & \text{otherwise.} \end{cases}$$

$$\mathbb{E}R = \sigma\sqrt{\frac{\pi}{2}}, \quad \operatorname{Var} R = \frac{4-\pi}{2}\sigma^2$$

$$f_R(r) = \begin{cases} \frac{1}{\sigma^2} r e^{-\frac{1}{2}\left(\frac{r}{\sigma}\right)^2}, & r > 0, \\ 0, & \text{otherwise.} \end{cases}$$





John William Strutt, 3rd Baron Rayleigh (1842 –1919)

- English physicist
- Discovered argon > Nobel Prize
- Discovered Rayleigh scattering, explaining why the sky is blue

Complex-Valued Random Variables

- Complex-valued random variable: $Z = X + jY$
- X and Y are real-valued random variable
- $\mathbb{E}Z = \mathbb{E}X + j\mathbb{E}Y$
- $\sigma_Z^2 \equiv \mathrm{Var}[Z] = \mathrm{Cov}[Z,Z] = \mathbb{E}\left[|Z - EZ|^2\right] = \sigma_X^2 + \sigma_Y^2$

$$\mathrm{Cov}[Z_1, Z_2] = \mathbb{E}\left[(Z_1 - \mathbb{E}Z_1)(Z_2 - \mathbb{E}Z_2)^*\right] = \mathbb{E}\left[Z_1 Z_2^*\right] - (\mathbb{E}Z_1)(\mathbb{E}Z_2)^*$$

- Suppose $Z = X + jY$ where $X, Y \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma^2)$
  We write $Z \sim \mathcal{CN}(0, \sigma_Z^2) = \mathcal{CN}(0, 2\sigma^2)$

$$f_Z(z) = f_{X,Y}(x,y) = \left(\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}\right)\left(\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}}\right) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} = \frac{1}{2\pi\sigma^2} e^{-\frac{|z|^2}{2\sigma^2}} = \frac{1}{\pi\sigma_Z^2} e^{-\frac{|z|^2}{\sigma_Z^2}}$$

Rayleigh Fading Channel

- $h \sim \mathcal{CN}\left(0,\sigma^2\right)$: $\quad \mathrm{Re}\{h\},\mathrm{Im}\{h\} \overset{\text{i.i.d.}}{\sim} \mathcal{N}\left(0,\frac{\sigma^2}{2}\right)$
  - Usually normalized so that $\sigma^2 = 1$

- $n \sim \mathcal{CN}\left(0,N_0\right)$: $\quad \mathrm{Re}\{n\},\mathrm{Im}\{n\} \overset{\text{i.i.d.}}{\sim} \mathcal{N}\left(0,\frac{N_0}{2}\right)$
- Most applicable when
  - there is no dominant propagation along a line of sight between the transmitter and receiver
    - If there is a dominant line of sight, **Rician fading** may be more applicable.
  - there are many objects in the environment that scatter the radio signal before it arrives at the receiver
- Ex. Densely-built Manhattan.

Digital Communication Systems ECS 452

Asst. Prof. Dr. Prapun Suksompong

prapun@siit.tu.ac.th

Introduction to Multiple-Antenna System

Office Hours:
Rangsit Library:
Tuesday 16:20-17:20
BKD3601-7:
Thursday 16:00-17:00

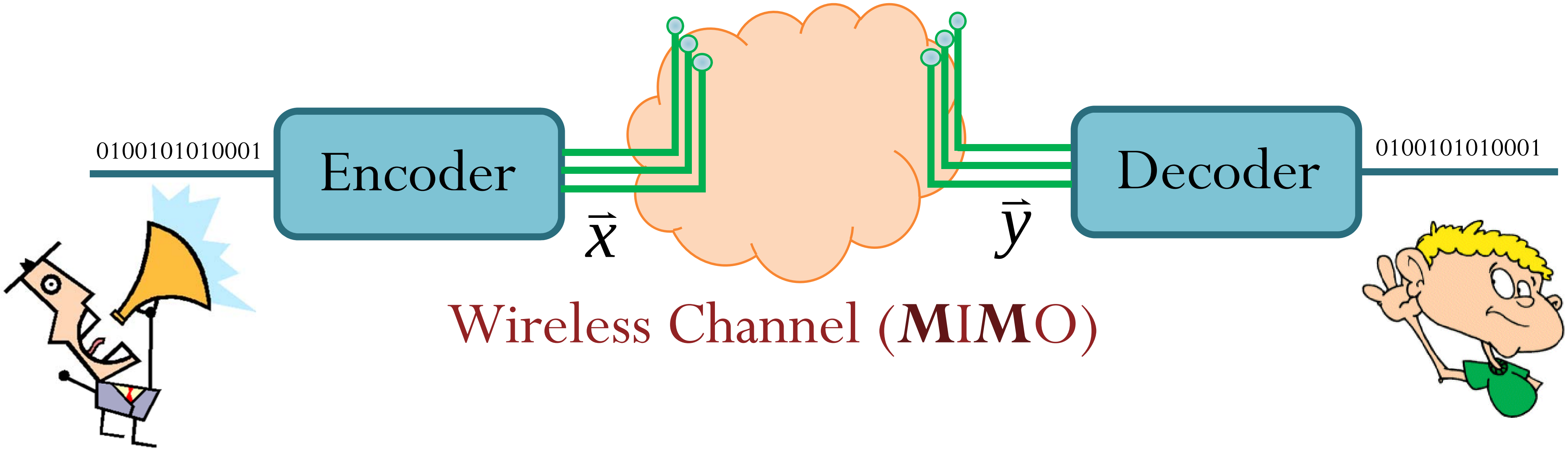
Multiantenna Systems

- Since the 1990s, there has been enormous interest in multiantenna systems.
- Two types [Molisch, 2011, p 445]
  - **Smart antenna systems**
    : multiantenna elements at one link end only
    - Ex. Rx smart antennas
      - Signals from different elements are combined by an adaptive (intelligent) algorithm
      - Intelligence (smartness) is not in the antenna, but rather in signal processing.
  - **Multiple Input Multiple Output (MIMO) systems**
    (Pronounced *mee-moh* or *my-moh*)

    :multiantenna elements at both link ends.

MIMO Channel Model

(Multiple Input Multiple Output)

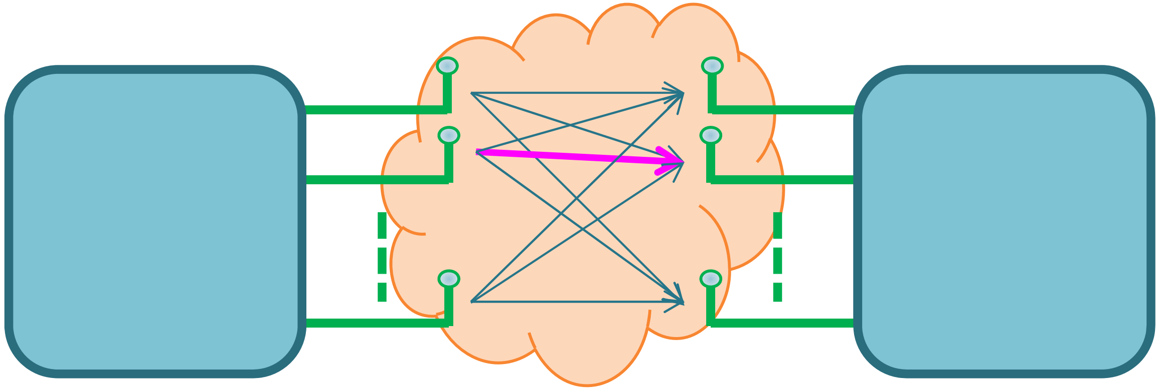


$$\vec{y} = \mathbf{H}\vec{x} + \vec{n}$$

Channel Matrix

Noise

MIMO Channel Model



$$\vec{x} \qquad\qquad \vec{y} = \mathbf{H}\vec{x} + \vec{n}$$

- **H** is now a matrix.
- Its entries form an i.i.d. Gausian collection with zero-mean, independent real and imaginary parts, each with variance ½.
- Equivalent, each entry of H has uniform phase and Rayligh magnitude.

$h_{i,j}$ = complex channel gain from the jth transmit to the ith receive antenna

$$H = \begin{bmatrix} h_{1,1} & h_{1,2} & \cdots & h_{1,N_T} \\ h_{2,1} & h_{2,2} & \cdots & h_{2,N_T} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_R,1} & h_{N_R,2} & \cdots & h_{N_R,N_T} \end{bmatrix}$$

From Impairment to Opportunity

- Multipath scattering is commonly seen as an **impairment** to wireless communication.
- However, it can now also be seen as providing an **opportunity** to significantly improve the capacity and reliability of such systems.
- By using **multiple antennas** at the transmitter and receiver in a wireless system, the **rich scattering channel** can be exploited to create a **multiplicity** of **parallel links** over the same radio band, and thereby
  - to either increase the **rate** of data transmission through (**spatial**) **multiplexing** (transmission of **several data streams in parallel**) or
  - to improve system **reliability** through the increased antenna **diversity**.
- Moreover, we need not choose between **multiplexing and diversity**, but rather we can have both subject to a fundamental **tradeoff** between the two.

MIMO Benefits: Spatial Diversity

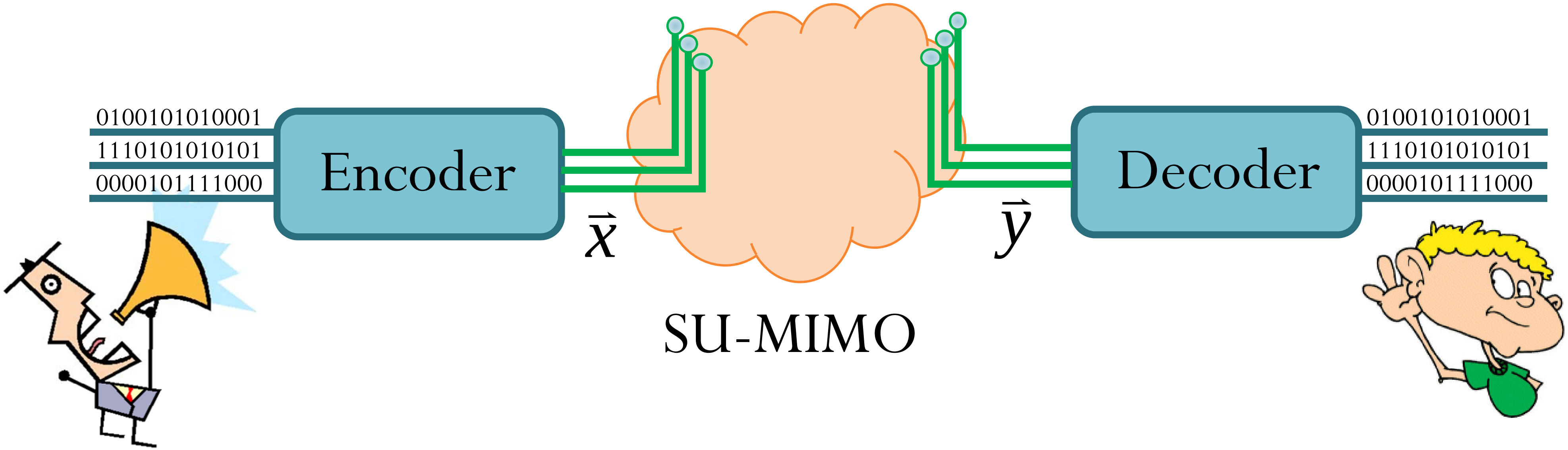
- Mitigates fading
- Realized by providing the receiver **with multiple (ideally independent) copies** of the transmitted signal in space, frequency or time.
  - With an increasing number of independent copies (the number of copies is often referred to as the **diversity order**), the probability that at least one of the copies is not experiencing a deep fade increases, thereby improving the quality and reliability of reception.
- A MIMO channel with N_T transmit antennas and N_R receive antennas potentially offers $N_T N_R$ independently fading links, and hence a **spatial diversity order of $N_T N_R$**.
- Improve **reliability**.

MIMO Benefits: Spatial Multiplexing

- MIMO systems offer a **linear increase** in data rate through spatial multiplexing, i.e., transmitting **multiple, independent data streams** (not multiple copies as in obtaining spatial diversity) within the bandwidth of operation.
  - Under suitable channel conditions, such as **rich scattering** in the environment, the receiver can separate the data streams.
  - Furthermore, each data stream experiences at least the same channel quality that would be experienced by a SISO system, effectively **enhancing the capacity** by a multiplicative factor equal to the number of streams.
- In general, the number of data streams that can be reliably supported by a MIMO channel equals $\min\{N_T, N_R\}$.

MIMO Benefits: Spatial Multiplexing

Transmit **multiple** independent data **streams** or spatial streams on different antennas



Problem: Interference among transmitting antennas

Solution: **Pre-process** (pre-code) the transmitted signals

MIMO Coding Schemes

- Achieve the best spatial diversity:
  - **space-time trellis codes**
  - **space-time block codes**
- Maximize the transmission rate:
  - **Bell Lab layered space-time** (**BLAST**) coding schemes
- These two families of space-time codes represent two extremes in the sense that one achieves the best reliability and the other achieves the maximum transmission rate.
- Other space-time coding schemes that provide a trade-off between diversity and rate also exist.

Ex. Spatial Multiplexing

$$\vec{y} = \mathbf{H} \underbrace{\mathbf{A}\vec{s}}_{\vec{x}} + \vec{n}$$

conjugate transpose

If **H** can be **decomposed** as

$$\mathbf{H} = \mathbf{Q}\tilde{\mathbf{H}}\mathbf{P}^H, \quad \text{with } \mathbf{Q}^H\mathbf{Q} = \mathbf{P}^H\mathbf{P} = \mathbf{I}$$

and we set $\mathbf{A} = \mathbf{P}$, then at the receiver, we have

$$\vec{y} = \mathbf{Q}\tilde{\mathbf{H}}\mathbf{P}^H\mathbf{A}\vec{s} + \vec{n} = \mathbf{Q}\tilde{\mathbf{H}}\mathbf{P}^H\mathbf{P}\vec{s} + \vec{n} = \mathbf{Q}\tilde{\mathbf{H}}\vec{s} + \vec{n}.$$

Ex. Spatial Multiplexing

Finally, we can find

$$\vec{r} = \mathbf{Q}^H \vec{y} = \mathbf{Q}^H \mathbf{Q} \tilde{\mathbf{H}} \vec{s} + \mathbf{Q}^H \tilde{n} = \tilde{\mathbf{H}} \vec{s} + \tilde{n}$$

The whole MIMO system can be reduced to

$$\vec{r} = \tilde{\mathbf{H}} \vec{s} + \tilde{n}$$

Q: Why is this better than our original

$$\vec{y} = \mathbf{H} \vec{x} + \vec{n}$$

A: "Clever" decomposition

can **reduce** the **interference** among data streams.

Ex. Spatial Multiplexing

- Conventional scheme uses **SVD** (Singular Value Decomp.)
- Alternatively, we can use **GTD** (Generalized Triangular Decomposition)

[Jiang et al. 2004,2007]

SVD

$$\mathbf{H} = \mathbf{U}\mathbf{D}\mathbf{V}^H$$

$$\tilde{\mathbf{H}} = \mathbf{D} = \begin{bmatrix} D_{11} & 0 & 0 \\ 0 & D_{22} & 0 \\ 0 & 0 & D_{33} \end{bmatrix}$$

GTD

$$\mathbf{H} = \mathbf{Q}\mathbf{R}\mathbf{P}^H$$

$$\tilde{\mathbf{H}} = \mathbf{R} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ 0 & R_{22} & R_{23} \\ 0 & 0 & R_{33} \end{bmatrix}$$

Ex. Spatial Multiplexing

$$\vec{r} = \tilde{\mathbf{H}}\vec{s} + \tilde{n}$$

SVD

$$\tilde{\mathbf{H}} = \mathbf{D}$$

$$r_1 = D_{11}s_1 + \tilde{n}_1$$

$$r_2 = D_{22}s_2 + \tilde{n}_2$$

$$r_3 = D_{33}s_3 + \tilde{n}_3$$

Streams are completely separated

GTD

$$\tilde{\mathbf{H}} = \mathbf{R}$$

$$r_1 = R_{11}s_1 + R_{12}s_2 + R_{13}s_3 + \tilde{n}_1$$

$$r_2 = R_{22}s_2 + R_{23}s_3 + \tilde{n}_2$$

$$r_3 = R_{33}s_3 + \tilde{n}_3$$

Can use successive cancellation